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The Physics Behind Garage Door Springs

Tal Joseph
Madison Area Technical College

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Abstract

Pushing a button to open your garage door before going to work seems trivial until the motor fails to lift the door. There are various explanations for a garage door malfunction, but this paper will focus on the most common of them all—a broken spring. Inspired by the fact that garage door technicians must match the right spring to the appropriate garage doors, this project produced a spring conversion calculator. This paper provides a preface and comprehensive explanation of the mechanisms of overhead garage doors. Initially, it introduces the garage door and its components, then provides a detailed explanation of a door's lifting mechanisms, and lastly, elaborates on spring theory, focusing on the physics and calculations of spring properties.

Introduction

At approximately 7 to 8 feet tall and 9 to 16 feet wide, common residential garage doors, known as *overhead* doors, are large enough to fit one or two cars. The doors are typically made from four to five horizontal panels attached to one another by hinges. Garage doors are usually several hundred pounds, but they are relatively easy to lift manually because of one or more torsion springs, which are attached to rotating shafts above the doors. When a door is closed, the torsion springs are under tension, and the energy stored in the wound spring(s) does most of the work of raising the door.

There are six principal components of a garage door: cables, drums, torsion shaft, tracks, rollers, and springs (see Figure 1).

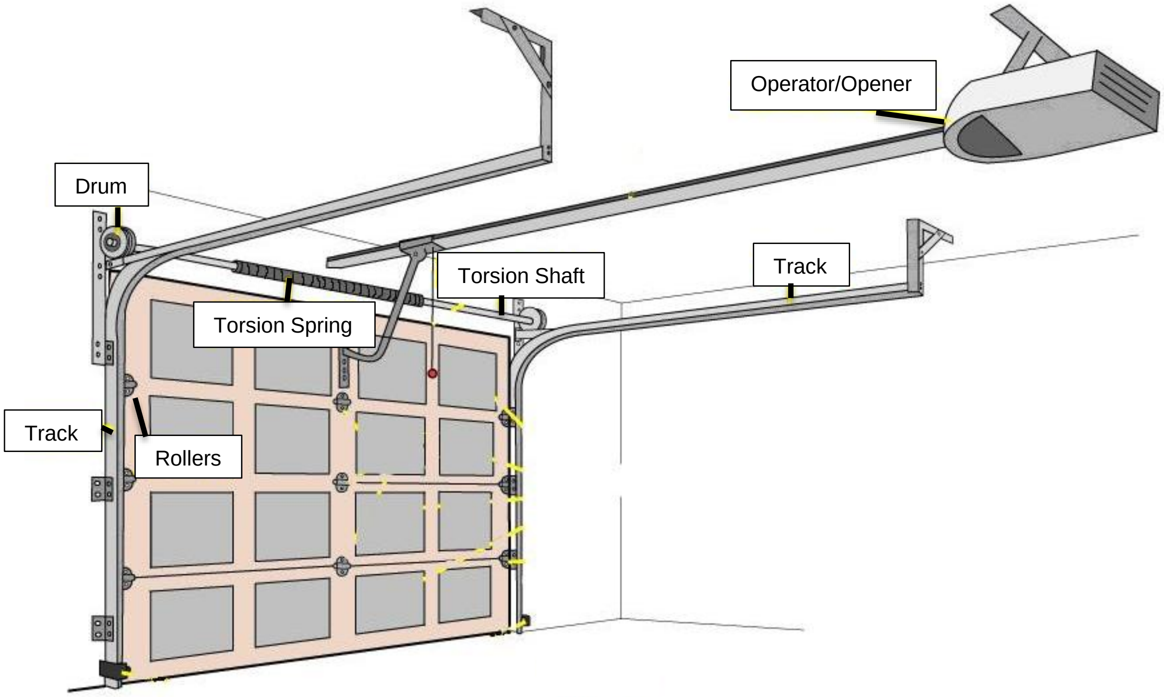

Figure 1. Garage Door Components. Adapted from *Popular mechanics*, by M. Iglesias, 2015, Retrieved April 11, 2017, from www.popularmechanics.com/home/outdoor-projects/how-to/a6041/garage-door-opener-how-it-works/. Copyright 2017 by Hearst Communications, Inc.

On each side of the door there is a thin cable that is connected to the bottom of the door and extends all the way to the drum above it (see Figure 2). The drums are pulleys with grooves that accommodate the cables. They sit on each side of a torsion shaft, which is a freely-rotating metal rod that runs horizontally across the top of the door. At the sides of the door are the rollers; wheel-like structures that keep the door aligned inside the side-tracks, and allow the door to freely roll up and down. The torsion spring is installed on the torsion shaft with one side bracketed to the wall of the garage and the other side locked against the torsion shaft via winding cones (see Figure 11).

While the door is closed and before the torsion spring is physically attached to the torsion shaft during installation, the spring is wound by rods *T* times to a torsion sufficient enough to generate an upward force equal to the weight of the door. For example, in the case of a 200-pound door with two torsion springs, each spring at maximum torsion should supply about 100 pounds of force. The spring is fully loaded when the door is closed; when the door is lifted, the spring unwinds and loses its power gradually. As the door lifts, the horizontal track compensates for the door's immense weight. Although the spring becomes gradually weaker with the opening of the door, the door's weight decreases as it moves vertically and horizontally, which enables the spring to continue pushing the door (see Figure 2).

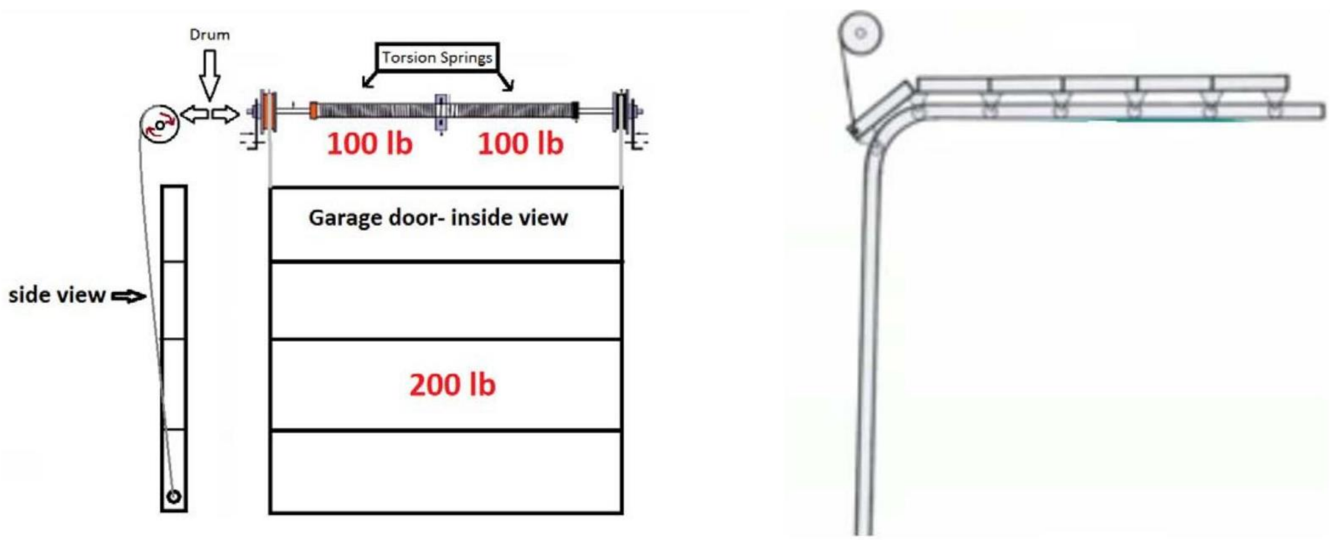

Figure 2. Garage Door Side and Inside Views. Left illustration was adapted from *Home and Dollars,* By Chris, 2016, Retrieved and modified April 11, 2017, from www.homeanddollars.com/2016/. Right illustration was adapted from *DDM Garage Doors,* Retrieved April 11, 2017, from ddmgaragedoors.com/diy-instructions/intro-to-counterbalance.php. Copyright 2016 by DDM Web Services. Inc.

Lifting Mechanisms

Before delving further into the calculations of the aforementioned lifting mechanism, alternative lifting mechanisms will be introduced. A *vertical-lift* garage door rises vertically and has no horizontal tracks (see Figure 3). Vertical-lift mechanisms are typical of industrial-sized garages, but they are uncommon for residential homes because it is unusual for a residence to have enough space for upward movement of the required height (7-8 ft). The specialized component of this mechanism is the cone-shaped drum (see Figure 4):

Figure 4. Cone-shaped Drum. Picture

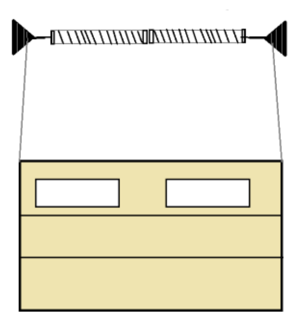

Figure 3. Vertical-Lift Door. Made with Microsoft-Paint.

The lifting capability of a spring is related to the diameter of the drum. A typical, evenly-shaped drum would only be able to lift a vertical-lift door a short distance because the spring would lose tension and the weight of the door remain constant, since there is no horizontal track to carry its weight. Special cone-shaped drums, compensate for the loss of spring tension and the constant weight of the door. When the spring starts spinning, the cables reels on the wider side of the drums, and approach the narrow sides as the door lifts (“Introduction to Garage Door Counterbalance”).

A *high-lift* garage door utilizes a third lifting mechanism. The high-lift mechanism is a hybrid mechanism that encapsulates qualities of the vertical lift and the over-head lift. In residences that have high ceilings, that are not high enough for the vertical-lift, high-lift garage doors – which lift higher than the typical over door garage door, but still rise horizontally—can be installed. The high-lift mechanism relies on a hybrid drum that combines elements from both previously discussed mechanisms (see Figure 5):

Figure 5. Hybrid Drum. Picture

Spring Geometry and Essential Theory

The revolution of a circle about an axis—under the condition that the revolution never intersects the original circle—generates a “donut” or “tire shape” called a torus (see Figure 6).

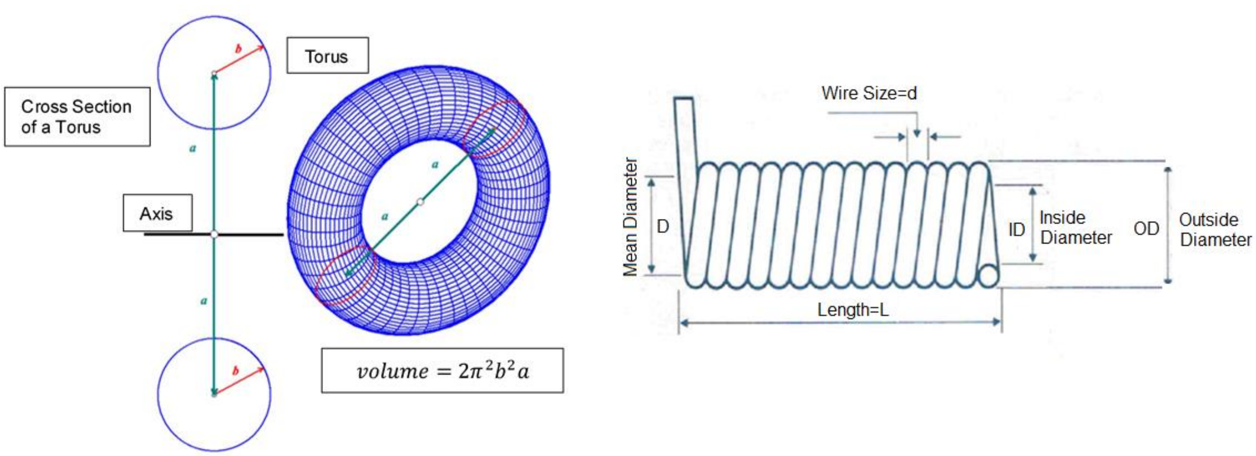

Figure 6. Spring Specs & Cross Section of a Torus. Left part was illustrated with Winplot. Right section was adapted from *Fox Valley Spring Company.* Retrieved April 12, 2017, from *www.foxvalleyspring.com/springs.php*. Copyright 2017 Fox Valley Springs.

The torus has a volume given by $V=2\pi^2 b^2 a$ (see appendix A,1). Because most springs in practice have a very small angle of pitch between coils, the spring can be modeled as a stacked column of closely spaced adjacent tori. The mean diameter, D, of the spring is twice the torus dimension *a* and the spring wire size, *d*, is twice the torus dimension *b* (see Figure 6).

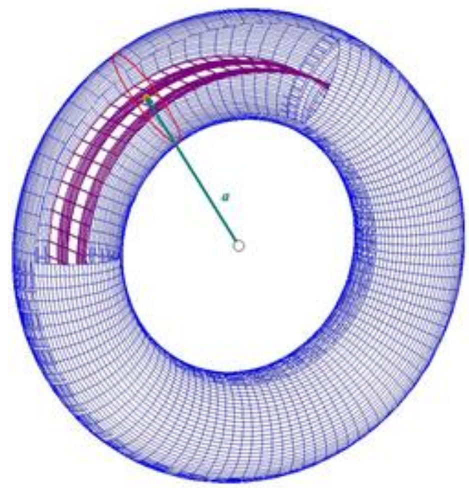

Figure 7. Neutral Surface in a Coil. Illustrated with Winplot.

Although a garage door springs are called a torsion spring, they do not work based on torsion—i.e., a torque generated by twisting a bar about its longitudinal axis. The torque—the ability to rotate an object – in a torsion spring results from curvature or bending. When a solid object is forced to bend around a center, stresses are generated on both sides of the solid (see Figure 7). The side on the outside diameter from the center is stretched and undergoes tension. In cross section near the side furthest from the center, there are forces that originate from the rest of the solid and tend to pull the solid apart. The side on the inner diameter of the solid closest to the center is shortened and experiences compressive stress. In cross section near the side closest to the center there are forces that originate from the rest of the solid and tend to push the solid inward.

Evidently, there must be a place within the solid where these internal stresses are neither pulling out nor pushing in. This area is the *neutral surface,* and in a torus, the neutral surface is a section of a cylinder of the radius, *a*, and height, $2b = d$ (see Figure 7). The coil is strained at any point in a circular cross section that does not lie on the neutral surface. The amount of strain at a distance *y* from the wire center—the neutral surface— due to bending around the center is given by $\frac{y}{a}$. The stress experienced at this point is given by $S = \frac{Ey}{a} = \frac{E2y}{D}$ and is directed perpendicular to the circular cross section. *E* represents Young's Modulus of Elasticity, which measures the stiffness of a solid material, and relates stress to strain. The stress is positive regarding tension for $y > 0$ and negative regarding compression for $y < 0$. (see Figure 8).

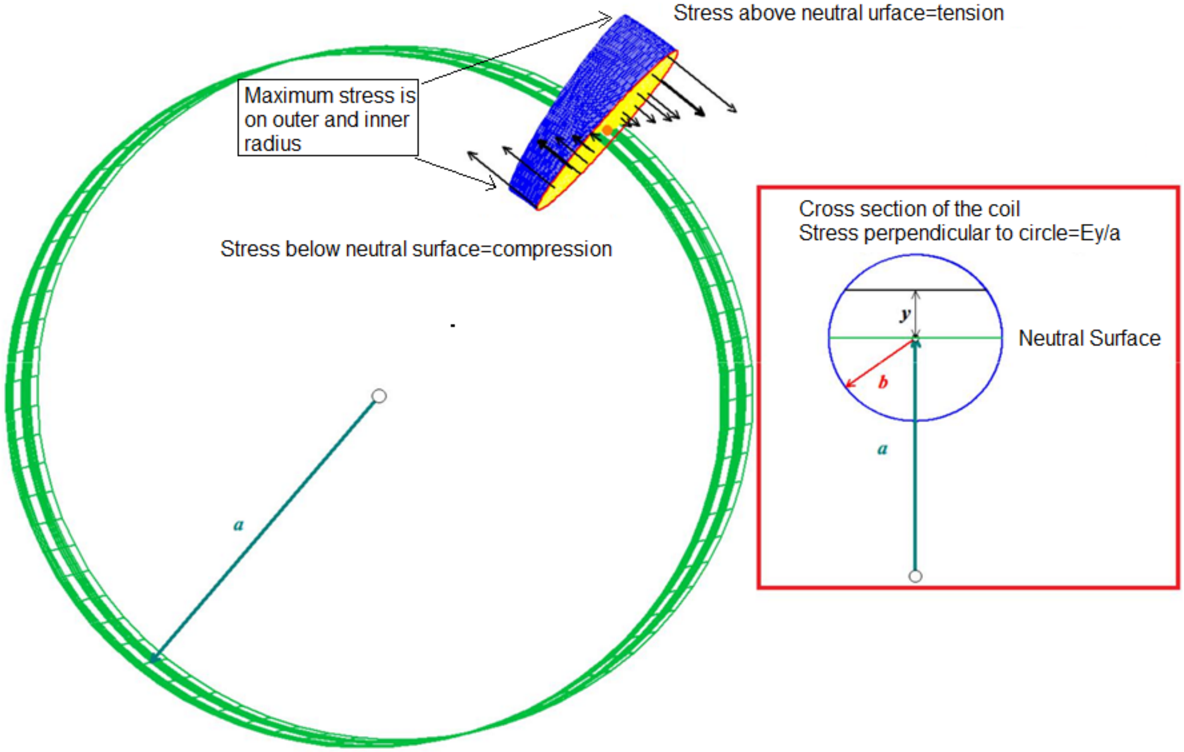

Figure 8. Stresses Above and Under the Neutral Surface. Illustrated with Winplot.

The greatest stress due to bending occurs when $y = b$ and $y = -b$ and has a value of $S = \frac{Ed}{a}$. This stress is referred to as the *bending shear stress*, and fractures of a spring coil are most likely to begin at the inner and outer diameters of the bending shear (see Figure 9). Eventually, any torsion spring will fail after repeated use (see Figure 10).

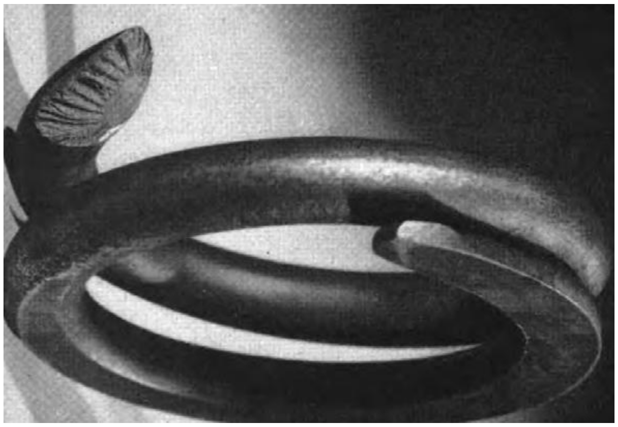

Figure 9. Typical Fatigue Failure. Adapted from *Mechanical Springs* (p. 31), by A.M Wahl, 1944,

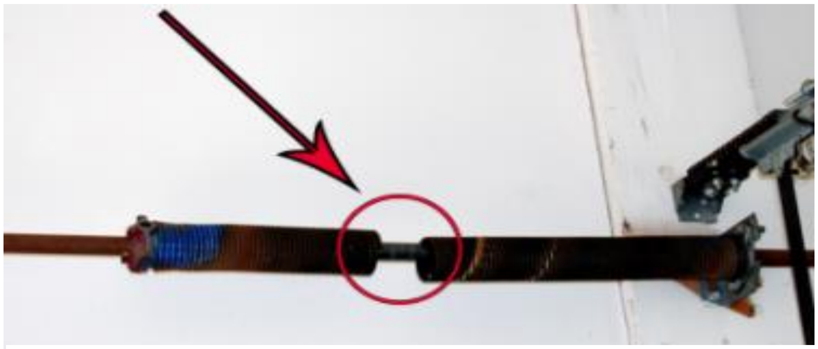

Figure 10. Broken Spring. Adapted from *Madison Local Garage Door Pros,* retrieved April 20, 2017, from *www.madisongaragerepair.com/garage-door-spring/*. 2017 by Local Garage Door Pros.

The stresses on a circular cross section of the wire are of opposite direction above and below the neutral surface, hence, they develop a torque that acts on each circular face due to the bending. The magnitude of this torque on one circular face of a single coil is given by $\tau = \frac{\pi E d^4}{32\text{D}}$ (see Appendix B). But this is balanced by a counter torque on the other side of the wire element, so the net torque acting on each circular wire-element is zero. The bending shear stress is expressed by the equation $S = \frac{Ed}{\text{D}} = \frac{32\tau}{\pi d^3}$. The unwound spring exerts no torque to the torsion shaft. However, when the spring is wound T turns, or full revolutions, by an external torque (due to technician rotating the winding cones), work is done on each active coil of the spring and the resulting energy is stored in the wound spring. The coils, which are pinned against the winding cones, are called *dead coils*, and the remaining coils are *active coils* which are represented by N (see Figure 11). The amount of energy per active coil for a single turn of the winding cones is given by $2\pi K$, where K is the spring rate given by $K = \frac{\pi E d^4}{32\text{D}N} = \frac{\tau}{N}$ (Wahl 1944). When the torsion shaft is free to rotate the energy stored in the spring generates a torque which causes the torsion shaft and the attached pulley to rotate. The rotation of the torsion shaft and the pulley pulls on the cables attached to the bottom of the closed door. The initial torque is computed as $\tau_i = KT = \frac{\pi E d^4 T}{32\text{D}N}$. Because torque is the product of force multiplied by a radius at perpendicular angles to the force, the initial lifting force exerted by the spring on the garage door is given by the following equation, $F = \frac{\tau_i}{r} = \frac{\pi E d^4 T}{32\text{D}N r}$, where r is the radius of the pulley attached to the torsion shaft. Due to friction, some additional force is required to fully lift the door. The additional force is applied by an automatic garage door opener or a human pulling up on the handle within a manual system.

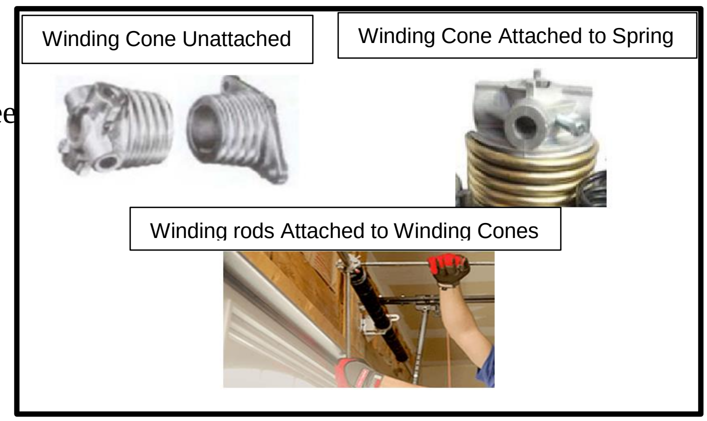

Figure 11. Winding Cones & Winding Rods. Adapted from Madison Local Garage Door Pros, by staff. Retrieved April 20, 2017, from *www.madisongaragerepair.com/2017/01/03/garage-door-springs/*. Copyright 2017 by Local Garage Door Pros.

Lastly, in order to discover the lifespan of a spring, the Wahl correction factor, W_c is used to obtain a more accurate estimate of the bending shear stress, $W_c S$. Table 1 summarizes the above equations and Table 2 provides an estimate of the spring's lifespan (Karwa 2006):

Table 1.

Summary of Equations

$V = 2\pi^2 b^2 a$	$K = \frac{\pi E d^4}{32DN} = \frac{\tau}{N}$
$\tau = \frac{\pi E d^4}{32D}$	$\tau_i = KT = \frac{\pi E d^4 T}{32DN}$
$S = \frac{32\tau}{\pi d^3}$	$F = \frac{\tau}{r}$
$W_c = \frac{4D-d}{4(D-d)} + \frac{0.615d}{D}$	$D = ID + d$
$N = \frac{L}{d} - \text{"dead coils"}$	$T\ (turns)$

Table 2.

Estimated Lifespan of a Spring

$W_c S < 242 \frac{10^3 lb}{in^2}$	~10,000 cycles
$W_c S < 200 \frac{10^3 lb}{in^2}$	~25,000 cycles
$W_c S < 175 \frac{10^3 lb}{in^2}$	~50,000 cycles
$W_c S < 150 \frac{10^3 lb}{in^2}$	~100,000 cycles

Note. Adapted from "Calculating Spring Properties" by Richard J. Kinch, 2015. Retrieved 20 April 2017 from www.truetex.com/garage.htm. Copyright 2015 by Richard J Kinch.

In the following calculations supplied is the common material torsion springs are made of— ASTM (American Society for Testing and Materials) A229 oil tempered steel wire which has a mean weight density of $\frac{0.282 \text{ lb}}{\text{in}^3}$ and a Young's modulus of $E = \frac{2.90 \times 10^7 \text{ lb}}{\text{in}^2}$ ("ASTM A229 Oil-tempered Steel Wire."). Additionally, the calculations are based on an average of five dead coils, which will be deducted from the total number of coils, according to the rule of thumb for residential size springs. All the cable on the pulley is effectively 2.00 inches from the center of the torsion shaft. Because the cable stretches very little, it needs to be lifted to the height of the door. The greater the value of *T (spring revolutions)*, the greater the torque delivered by the wound spring. There must be some force exerted on the cable by the spring when the door is fully raised in order to keep the cable on the pulley; therefore, adding one-fourth of a turn to the geometric estimate is necessary.

The calculation of the maximum force required to manually wind the spring is computed as follows: The maximum torque needed to wind the spring occurs at the end of the winding process and equals the maximum torque exerted by the spring. Using the garage door industry standard of 18-inch winding rods, which are inserted into the attached winding cones (see Figure 11), the maximum force is the maximum torque divided by 18 inches.

Calculations

The following is an example of calculations made for an arbitrary spring:

Spring Rate and Torque

Spring rate and torque: let's pick a spring with wire size d = 0.243 inches, length (L) of 30.5 inches, and ID of 2 inches. Its mean diameter D = 2.243 inches (ID+d=D). The number of coils is approx. L/d = 30.5 inches / 0.243 inches = 126 coils. 126 minus 5 dead coils, or 121 active coils (N), is taken into consideration. Thus, the spring rate is K = (π*2.9*10^7 * (0.243) ^4) / (32 * 121 * 2.243) = 36.6 in/lb. (K=τ/N). Winding 7.25 turns * 36.6 in/lb. produce a torque of approx. 265 in/lbs. per spring.

Lifting Weight

The 4' lift drums have a radius of 2', so the lift of one spring is 262/2 = 131 lbs.

Stress and lifetime

Calculating the maximal stress of the spring's wire will assist us to estimate the lifetime of the spring. The bending stress S in the spring wire is 32*265/(π*0.243^3) = 188 Kpsi. The Wahl correction factor is W_c = (4*2.243-0.243)/ [4*(2.243-0.243)] + 0.615*0.243/2.243 = 1.1578 and the Wahl-corrected stress is W_c * S = 1.1578 * 188 Kpsi = 218 Kpsi. This predicts about a 15,000-cycle lifetime.

Appendix:

Mathematical Derivations

A. Volume of a Torus

Make a cut at a distance *r*, with $a - b \le r \le a + b$, inside the circle of radius *b* whose center is a distance *a* above the axis of rotation (see figure 12). Let *dr* = the thickness of the cut. Upon rotation of the circle about the axis this cut generates a cylindrical shell of radius *r*, height 2*x* and thickness *dr*. From the Pythagorean Theorem $x = \sqrt{b^2 - (r-a)^2}$. The element of volume of the shell is $dV = 4\pi r\sqrt{b^2 - (r-a)^2}\,dr$. Integrating this expression over the domain of *r* gives the volume of the torus.

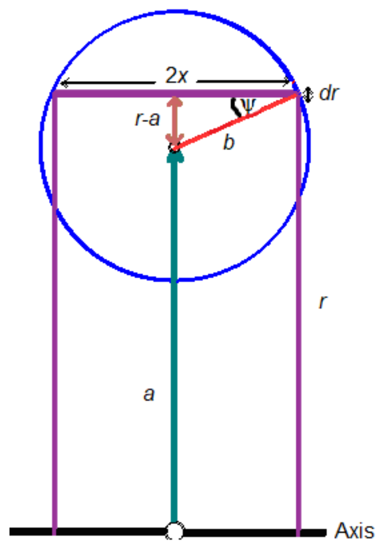

Figure 12. Cross Section of a Torus. Illustrated with Winplot.

$$V = \int_{a-b}^{a+b} 4\pi r\sqrt{b^2 - (r-a)^2}\,dr = 4\pi\int_{-b}^{b} (y+a)\sqrt{b^2 - y^2}\,dy$$

with $y = r - a$.

The trigonometric substitution $y = b\sin(\psi)$, results in the relations that

$\psi = \sin^{-1}\left(\frac{b}{a}\right)$, $\sqrt{b^2 - y^2} = b\cos(\psi)$, and $dy = b\cos(\psi)\,d\psi$.

The volume is now expressed as:

$$V = 4\pi\int_{-\pi/2}^{\pi/2}\left[b^3\sin(\psi) + ab^2\right]\cos^2(\psi)\,d\psi = \frac{-4\pi b^3}{3}\cos^3(\psi)\Big|_{-\pi/2}^{\pi/2} + 4\pi b^2 a\int_{-\pi/2}^{\pi/2}\frac{1+\cos(2\psi)}{2}\,d\psi,$$

where from the half angle formula $\cos^2(\psi) = \frac{1-\cos(2\psi)}{2}$. Since $\cos\left(\frac{\pi}{2}\right) = \cos\left(-\frac{\pi}{2}\right) = 0$, the volume of the torus is given by the following formula

$$V = 2\pi b^2 a\left[\psi + \frac{1}{2}\sin(2\psi)\right]\Bigg|_{-\pi/2}^{\pi/2} = 2\pi b^2 a\left(\frac{\pi}{2} + 0 - \frac{-\pi}{2} - 0\right) = 2\pi^2 b^2 a \quad \text{(Thomas 2005)}$$

This result can also be derived quite easily by using the second centroid theorem of Pappus (Thomas 2005).

B. The strain parallel to the neutral surface for pure bending of a single coil (see figure 13).

Consider the arc QP which is a distance $a+y$ from the center of the coil with $-b \le y \le b$. The length of QP is $(a+y)\Delta\theta$ where $\Delta\theta$ is the central angle measured in radians. If $y > 0$ QP is stretched compared to the arc $\mathrm{AB} = a\Delta\theta$ on the neutral surface, while if $y < 0$ QP is compressed compared to AB. The strain of QP is therefore $\dfrac{(a+y)\Delta\theta - a\Delta\theta}{a\Delta\theta} = \dfrac{y}{a}$ (Shigley, Joseph, and Mischke 2011). The resulting stress is perpendicular to the circular cross section and is directed out (tension) if $y > 0$ and is directed in (compression) if $y < 0$. From Hook's Law which states a linear relation between force and strain from equilibrium, the stress is given by $S = \dfrac{Ey}{a}$, where E is the Young's modulus of elasticity.

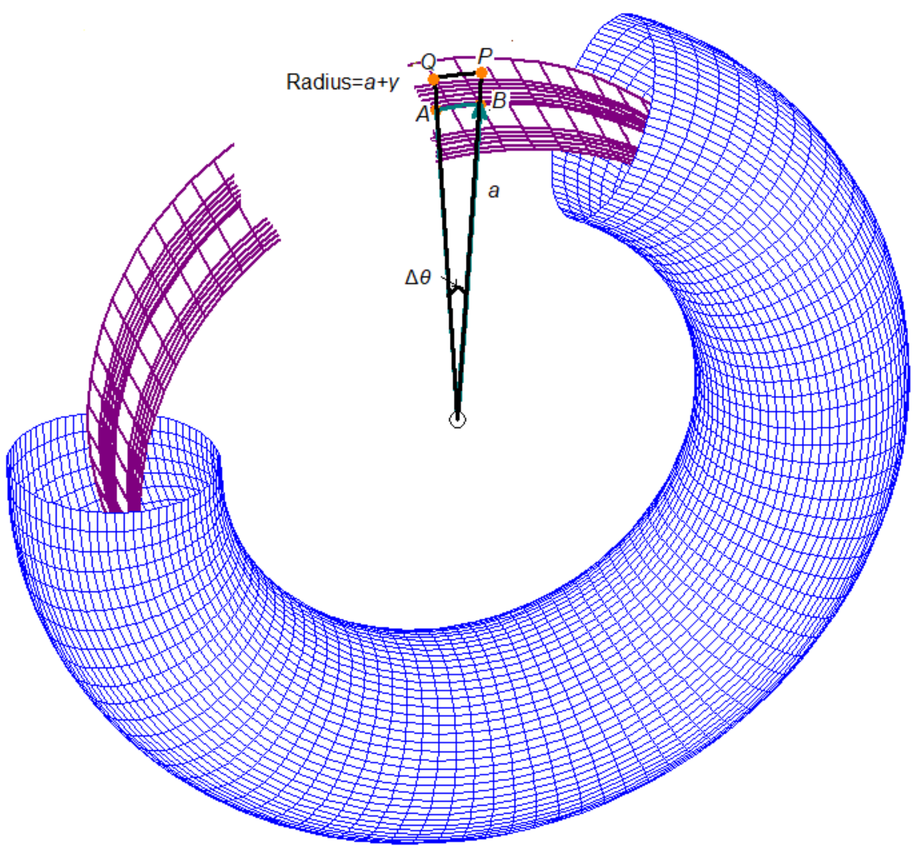

Figure 13. The Neutral Surface of a Coil. Illustrated with Winplot.

C. The moment for pure bending of a single coil (see figure 14).

Since the stress is of opposite direction above and below the neutral surface there is a bending moment or torque generated against the face of the circular element. The element of force on a strip with a vertical displacement of *y* is $dF = SdA$, where *dA* is the area of the strip with a vertical coordinate of *y* on a circle, $x^2 + y^2 = b^2$, centered at the origin. Expressing this strip as length times width gives $dA = 2xdy = 2\sqrt{b^2 - y^2}dy$, hence $dF = 2Sxdy = \frac{2}{a}Exydy = \frac{2}{a}Ey\sqrt{b^2 - y^2}dy$. The element of bending moment is given by $d\tau = ydF = \frac{2}{a}Ey^2\sqrt{b^2 - y^2}dy$.

Assuming that the material is uniform so that the Young's modulus is a constant, the bending moment against the circular face due to stress from the rest of the solid is given by the following definite integral. $\tau = \int_{-b}^{b} \frac{2}{a}Ey^2\sqrt{b^2 - y^2}dy = \frac{4E}{a}\int_0^b y^2\sqrt{b^2 - y^2}dy$.

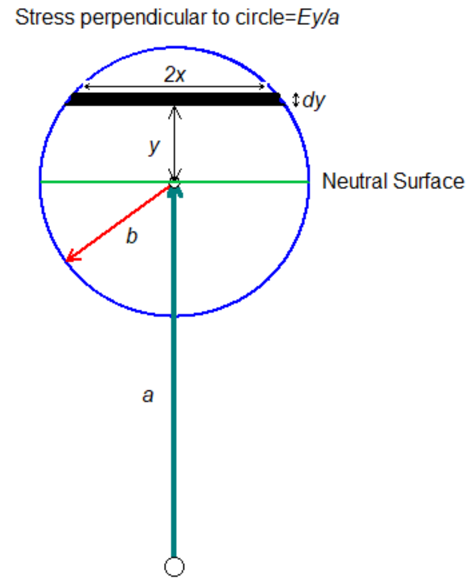

Figure 14. Cross Section of the Coil. Illustrated with Winplot.

Using the same trigonometric substitution of $y = b\sin(\psi)$ so that $\psi = \sin^{-1}\left(\frac{y}{b}\right)$, $dy = b\cos(\psi)d\psi$, and $\sqrt{b^2 - y^2} = b\cos(\psi)$, the moment becomes $\tau = \frac{4Eb^4}{a}\int_0^{\pi/2}\sin^2(\psi)\cos^2(\psi)d\psi$. From the double angle and half angle formulas: $\sin^2(\psi)\cos^2(\psi) = \left(\frac{\sin(2\psi)}{2}\right)^2$ and $\sin^2(2\psi) = \frac{1-\cos(4\psi)}{2}$, so the moment integral can be evaluated as $\tau = \frac{Eb^4}{2a}\int_0^{\pi/2}\left[1-\cos(4\psi)\right]d\psi = \frac{Eb^4}{2a}\left[\psi - \frac{\sin(4\psi)}{4}\right]\Bigg|_0^{\pi/2} = \frac{\pi Eb^4}{2(2a)}$. Finally, using that D = 2*a* and *d* = 2*b* the formula for the bending moment of a single coil, $\tau = \frac{\pi Ed^4}{32\mathrm{D}}$, is obtained (Wahl 1944).

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Addendum: A specific graphing software, provided by the college, was used to generate all figures displayed in this article. The graphing software that was used is the newest version of WinPlot©, which can be downloaded from MATC’s web site: faculty.madisoncollege.edu/alehnen/winptut/Install_Winplot.html.